Renewable and Sustainable Energy Reviews 40 (2014) 820–825

Contents lists available at ScienceDirect

Renewable and Sustainable Energy Reviews

journal homepage: www.elsevier.com/locate/rser

Comparative study of numerical methods for determining Weibull parameters for wind energy potential

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A R T I C L E I N F O

Article history:
Received 20 December 2013
Received in revised form
22 July 2014
Accepted 5 August 2014

Keywords:
Weibull distribution
L-moment method
Monte Carlo
Wind speeds
Wind energy

A B S T R A C T

Weibull distribution has been one of the most widely used distribution to determine potential of wind energy. Many different numerical methods can be used to estimate the parameters of the Weibull distribution. The L-moment method (L-MoM), which has not been used extensively in the previous literature about wind energy for the estimation of wind speed parameters relevant to the Weibull distribution has been presented and this method has been compared to the Moment method (MoM) and Maximum Likelihood (ML) method. Monte Carlo simulation has been used to compare the methods used in the estimation of the shape (k) and scale (c) parameters for a Weibull distribution. Moreover, MoM, L-MoM and ML parameter estimation methods have been used in analyzing an actual data set. Wind power densities have also been calculated with the help of estimated parameter values. We showed that, distribution is skewed to the right or is symmetrical and $n \geq 100$ the ML method is preferable in comparison to other methods in the estimation of the shape (k) parameter. The L-MoM method which we presented in this study may be beneficial for research using small sample sizes.

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1. Introduction

Energy, which is a determining indicator of the level of development of countries and one of the most important factors not only in industry but also in the daily life of the public, is obtained primarily from oil and natural gas. The sun, wind, and water also provide renewable sources of energy [1]. Fossil fuels play a significant role in meeting the demand for energy worldwide. However, considering the daily diminishing sources of fossil fuels and the environmental damage they cause, as well as the increasing price of and demand for them, interest in renewable energy resources continues to increase [2,3]. Energy obtained from the wind, which is considered among renewable energy resources, are an alternative to fossil-based fuels that has recently become popular as a source of energy [4]. Three important reasons exist for the increase in the generation of energy from wind. These are i) climate change, concern regarding fuel emissions, and public awareness regarding the relationship between energy sources and environmental issues; ii) the decrease in fossil oil and gas reserves and predictions regarding the inability to meet future demands

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http://dx.doi.org/10.1016/j.rser.2014.08.009
1364-0321/

from these reserves; and iii) the decrease in the cost required for windmills to produce wind energy, along with constant advances in technology [5–7]. The overall wind energy produced worldwide in 2011 was 237.660 MW. Of this established force, 96.606 MW is attributed to the European continent. As for Turkey, the total established force is only 1799 MW [8]. However, Turkey is in a good position with its 83,000 MW of technical potential for wind force [9]. In spite of this, more than half of Turkey's total energy consumption is provided via import. This ratio is expected to increase in the near future. The energy that will be required for development in Turkey will be obtained from renewable energy resources instead of fossil fuels. For this reason, wind force will play a crucial role as a renewable energy alternative.

The probability density function related to wind speed is vital information in wind energy implementations. Various probability density functions have been proposed to describe the distribution of wind speed. However, the two-parameter Weibull distribution is widely utilized in wind energy calculations [2,5,10–15]. Moreover, according to the International Standard (International Standard IEC 614-00-12), the two-parameter Weibull distribution has been deemed highly appropriate for wind speed data [16,17].

In the analysis of wind speed data, the distribution to which the data fit statistically should first be determined, and the relevant parameter estimations concerning this distribution should then be made. For the distributional fitting of wind speed data Anderson Darling, Chi-square and Kolmogorov–Smirnov tests can be performed. When fitting the data according to the Weibull distribution, various methods to estimate the distribution parameters can also be utilized [2,3,10,15,18,19]. The moment method, maximum likelihood method and graphical method are the most widely used methods. In the related literature, various methods have been compared, and relevant proposals have been discussed. However, the recommended methods and proposals vary based on the sample size, and the results of distribution fitting tests [2].

To calculate the wind energy potential, long-term meteorological observations will be required. When selecting an appropriate wind energy transformation system and suitable area, sometimes we have limitations (money, time, etc.) so we have to take short-term meteorological observations. When we have small sample size (short-term meteorological observations), L-moment method is more efficient rather than Maximum likelihood method and Moment method which are widely use in the literature (please see Table 1). The purpose of this particular study is to introduce the less widely known L-moment estimation method, which is used in estimation of two-parameter Weibull distribution parameters concerning wind speed, and to compare it with other methods. For this purpose, the methods of concern have been compared using a Monte Carlo simulation and an actual sample data set. The data used in the study, which were collected from Bilecik province in Turkey, were gauged at 10 meters on an hourly basis in the summer of 2008, and were obtained from the Republic of Turkey's Meteorology General Directorate. Used wind speed data in this study are obtained from Turkish Republic General Directorate of Meteorology which is the authorized official institution in the field of monitoring and collecting meteorological events.

This study consists of six subsections. The first describes the related general literature. The second and the third examine the Weibull distribution and the parameter estimation methods. The fourth presents the realized simulation and its results. The fifth compares the parameter estimation methods used. The final section summarizes and discusses the findings of the study.

Table 1
Simulation results.

n	$k=1.0$			$c=1.0$		
	MoM	L-MoM	ML	MoM	L-MoM	ML
30	0.036763	*0.027055*	0.028081	0.037784	*0.036634*	0.036687
100	0.010455	0.007429	*0.007036*	0.012117	0.011579	*0.011435*
500	0.002296	0.001758	*0.001627*	0.002657	0.002653	*0.002640*
1000	0.001011	0.000841	*0.000717*	0.001394	0.001395	*0.001328*
5000	0.000230	0.000131	*0.000111*	0.000177	0.000162	*0.000154*
10000	0.000081	0.000028	*0.000028*	0.000108	0.000107	*0.000112*
	$k=2.0$			$c=1.0$		
30	0.110161	*0.101895*	0.110776	0.009333	*0.009291*	0.009298
100	0.025678	0.025736	*0.025521*	0.002695	0.002700	*0.002693*
500	0.005956	0.006123	*0.005792*	0.000591	0.000592	*0.000590*
1000	0.001836	0.001719	*0.001698*	0.000332	0.000332	*0.000331*
5000	0.000562	0.000606	*0.000563*	0.000060	0.000060	*0.000059*
10000	0.000155	0.000178	*0.000164*	0.000043	0.000043	*0.000043*
	$k=3.4$			$c=1.0$		
30	0.324506	*0.300876*	0.313545	*0.003108*	*0.003108*	0.003108
100	0.075273	0.078167	*0.072598*	0.000921	0.000920	*0.000917*
500	0.017064	0.017757	*0.016356*	0.000210	0.000210	*0.000209*
1000	0.006680	0.006952	*0.006574*	0.000088	0.000088	*0.000088*
5000	0.001649	0.001834	*0.001274*	0.000014	0.000013	*0.000013*
10000	0.000322	0.000440	*0.000192*	0.000010	0.000009	*0.000009*
	$k=6.0$			$c=1.0$		
30	1.175033	*1.007468*	1.033168	0.001049	*0.001043*	*0.001043*
100	0.276899	0.254690	*0.231008*	*0.000305*	*0.000302*	*0.000302*
500	0.054785	0.052821	*0.044450*	*0.000063*	*0.000063*	*0.000064*
1000	0.028277	0.028499	*0.020756*	*0.000027*	*0.000027*	*0.000027*
5000	0.006821	0.006460	*0.005929*	*0.000008*	*0.000008*	*0.000008*
10000	0.000846	0.000669	*0.000622*	*0.000003*	*0.000003*	*0.000003*

2. The Weibull distribution

The two-parameter Weibull distribution has been widely used in the modeling of wind speed data. The reasons to use Weibull distribution are as follows: It fits the wind distribution very well; it has a flexible structure, varying according to the shape parameter of the distribution; it provides easy determination of parameters; the number of parameters is few; and once the parameters for a certain height are determined, the wind data for various heights can be calculated using the already determined parameters [2,20,21]. The two-parameter Weibull distribution consists of parameters with the same scale unit (c) as the wind speed and a dimensionless shape (k). The probability density function of the two-parameter Weibull distribution is as follows [22]:

$$f(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} \exp\left\{-\left(\frac{v}{c}\right)^{k}\right\}, \quad k>0, v>0, c>0. \tag{1}$$

in this formula: v stands for observed wind speed data, $f(v)$ is the probability related to the observed wind speed data, k represents the shape parameter, and c represents the scale parameter. The shape parameter (k) is a parameter indicating the wind frequency. This parameter is large when there is low variation in wind speed in a particular field. The scale parameter (c) represents relative cumulative wind speed frequency. In other words, the scale parameter changes according to the average speed. When the average speed is high, the scale parameter (c) is large [20].

The cumulative function can be obtained by calculating the integral of the probability density function presented in (1). The cumulative function relevant to a two-parameter Weibull distribution can be represented as follows [4].

$$F(v) = 1 - \exp\left\{-\left(\frac{v}{c}\right)^{k}\right\} \tag{2}$$

3. Parameter estimation methods

In this section, the Moment, L-Moment, and Maximum likelihood parameter estimation methods are presented.

3.1. The method of moment estimation (MoM)

In a general sense, the moment methodology is based on obtaining the unknown parameters out of equanimity by equalizing the theoretical moments related to the distribution and the moments obtained from the sampling. To estimate two-parameter Weibull distribution parameters using the moment method, the initial, coefficient of variation related to the sample should be calculated. The shape parameter is calculated by equalizing the calculated sample coefficient of variation and the theoretical coefficient of variation. Coefficient of variation related to the sample is presented as follows:

$$\hat{CV}_{MoM} = \left[\frac{\left(\sum_{i=1}^{n} v_i^2 \right) n}{\left(\sum_{i=1}^{n} v_i \right)^2} - 1 \right] \tag{3}$$

in Eq.(3), v_i stands for the wind speed data and n stands for the sample size.

The $\hat{k}$ value is calculated through the CV value obtained from the sample using the following equation.

$$\left\{ \frac{\Gamma\left(\frac{2}{\hat{k}}+1\right)}{\left[\Gamma\left(\frac{1}{\hat{k}}+1\right)\right]^2} - 1 \right\} = \hat{CV}_{MoM} \tag{4}$$

The calculated $\hat{k}$ value is the estimation of shape parameter obtained with the MoM method for the two-parameter Weibull distribution.

In Eq. (4), Γ represents the gamma function. In the third step, the estimation of scale parameter (c) is calculated using the shape parameter (k) estimated via the moment method. To estimate the scale parameter, the following equation can be utilized.

$$\hat{c} = \left[\frac{\left(\frac{1}{n}\right) \left(\sum_{i=1}^{n} v_i \right)}{\Gamma\left(\frac{1}{\hat{k}}+1\right)} \right] \tag{5}$$

in Eq. (5), v_i stands for the wind speed data and n stands for sample size.

3.2. The L-moment estimation method (L-MoM)

L-moment is an estimation method based on the linear combination of order statistics [23,24]. This estimation method is efficient and yields robust estimators against outliers [25].

Let X be a random variable with a cumulative distribution function given by (2), and let $X_{1:n} \le X_{2:n} \le \ldots \le X_{n:n}$ be the order statistics of a random sample of size n from the distribution of X.

The L-moments are defined to be the quantities

$$\lambda_r = \frac{1}{r} \sum_{j=1}^{r-1} (-1)^j C_{r-1}^{j} E(x_{r-j:r}) \tag{6}$$

for $r=1, 2, \ldots, n$, where $C_n^r = \frac{n!}{r!(n-r)!}$. The coefficient of L-variation (CV_{L-MoM}) is defined as follows:

$$CV_{L-MoM} = \frac{\lambda_2}{\lambda_1} \tag{7}$$

For a data sample of size n, the data are first sorted in the ascending order of $X_1 \le X_2 \le \ldots \le X_n$. Then, the unbiased estimator of CV_{L-MoM} is calculated using the following formulas:

$$\hat{CV}_{L-MoM} = \left\{ \frac{\left(\frac{2}{n}\right) \left[\sum_{i=1}^{n} \left(\frac{i-1}{n-1}\right) x_i \right]}{\left(\frac{1}{n}\right) \left[\sum_{i=1}^{n} x_i \right]} - 1 \right\} \tag{8}$$

With the calculated coefficient variation related to the sample then,

$$\left\{ 1 - \left[2^{-\left(\frac{1}{\hat{k}}\right)} \right] \right\} = \hat{CV}_{L-MoM} \tag{9}$$

the $\hat{k}$ value is calculated. The calculated $\hat{k}$ value is the L-MoM estimation of the shape parameter for the two-parameter Weibull distribution. After the estimation of shape parameter k with the L-MoM method by using Eq. (9), the scale parameter (c) can be estimated using Eq. (10).

$$\hat{c} = \left[\frac{\left(\frac{1}{n}\right) \sum_{i=1}^{n} x_i}{\Gamma\left(\frac{1}{\hat{k}}+1\right)} \right] \tag{10}$$

3.3. The maximum likelihood estimation method (ML)

The maximum likelihood method is a widely used prediction method in statistical estimation theory. Maximum likelihood methodology is based on the maximization of the likelihood function. However, ML estimations of the shape parameter for the two-parameter Weibull distribution can be calculated iteratively. For this reason, the iteration problem should be solved. The parameter that yields Eq. (11) is the k value of the ML estimation of shape parameter. This is found after calculation of the derivative for the shape parameter of the likelihood function.

$$\left\{ \left[\left(\frac{\sum_{i=1}^{n} v_i^{\hat{k}} \ln(v_i)}{\sum_{i=1}^{n} v_i^{\hat{k}}} \right) - \left(\frac{\sum_{i=1}^{n} \ln(v_i)}{n} \right) \right] - \hat{k} = 0 \right\} \tag{11}$$

The modified Newton–Raphson method has been used to solve Eq. (11). After the estimation of the shape parameter, the ML estimation of the scale parameter is determined using Eq. (12).

$$\hat{c} = \left(\frac{\sum_{i=1}^{n} v_i^{\hat{k}}}{n} \right)^{\left(\frac{1}{\hat{k}}\right)} \tag{12}$$

4. Monte Carlo simulation

In this study, a Monte Carlo simulation has been used to compare the methods used in the estimation of the shape (k) and scale (c) parameters for a Weibull distribution. In the simulations, the sample sizes are 30, 100, 500, 1000, 5000 and 10,000**;** the shape parameter values are 1.0, 2.0, 3.4, and 6.0; and the scale parameter value is 1.0. Fig. 1 shows graphics of the probability density function related to the two-parameter Weibull distribution for different shape parameter values.

To compare the estimation methods, the Mean Square Error (MSE) criteria have been utilized. For θ parameter, MSE criteria could be stated as follows:

$$MSE(\hat{\theta}) = E[(\hat{\theta} - \theta)^2] \tag{13}$$

Table 1 presents the simulation results obtained for shape and scale parameter values and for all sample sizes. As can be inferred from Table 1, the best estimation is yielded by the L-moment method in the prediction of a 30 unit sample size ($n=30$), with various shape, and scale parameters. It can be claimed that the ML method is the most effective method in the estimation of the shape parameter (k) when the sample size is 100 or more ($n \geq 100$). As for the estimation of the scale parameter (c), it can be claimed that there is no significant difference among the MoM, L-MoM and ML estimation methods. In cases in which the distribution is skewed to the right or is symmetrical and $n \geq 100$, the ML method is preferable in comparison to the other methods in the estimation of the shape (k) parameter.

5. Parameter estimation for actual wind data

In this study, the theoretically compared parameter estimation methods are applied to an actual wind data set. The data set, which belongs to Bilecik Province and was gauged at 10 m in the summer of 2008, was obtained from the Republic of Turkey's General Directorate of Meteorology. The data, though gauged at 10 m, was utilized to gauge the wind speed at 30 m via Hellman's coefficient of amplification. Hellman's coefficient of amplification can be stated as follows:

$$V_w = V_r \left(\frac{H_w}{H_r} \right)^{\alpha} \tag{14}$$

in the above equation, V_w represents the desired wind speed (in m/s); V_r represents the gauged wind speed (in m/s); H_w represents the desired height (in m); H_r represents gauge height; and α represents the surface situation [26]. The terrestrial coefficient of friction (α) is determined according to the surface situation. In this study, the value for α is 0.34. (For more information about α please see [27].)

The fitting of the data obtained at 10 m and 30 m to the two-parameter Weibull distribution has been tested via Anderson Darling (A^2). A^2 statistics can be calculated using the following formula [28].

$$A^2 = -N - \left\{ \sum_{i=1}^{n} \frac{2i-1}{n} [\ln F(v_i) + \ln(1 - F(v_{N+1-i}))] \right\} \tag{15}$$

A^2 statistic have been calculated to be 3.25 for the wind speed data gauged at 10 and 30 m. The p-Value related to the A^2 statistics is larger than 0.01 ($p > 0.01$). It can be claimed that wind speed data gauged at 10 and 30 m fit the Weibull distribution at a significance level of 0.01. Fig. 2 shows the distributional graphics relevant to the wind speed data described above.

Table 2 displays the parameter predictions obtained via the MoM, L-MoM and ML methods related to the wind speed data obtained in the summer of 2008 in Bilecik Province.

Here ($\hat{k}$) stands for the estimated shape parameter and ($\hat{c}$) represents the estimated scale parameter. The wind power density related to the wind speed can be calculated using the following

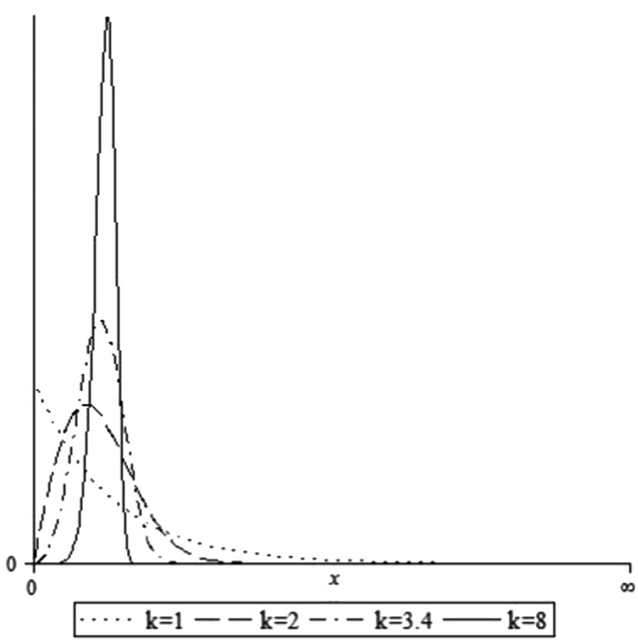

Fig. 1. Weibull distribution with different shape parameters.

Table 2
Parameter estimation for actual wind data.

	10 m		30 m	
	$\hat{k}$	$\hat{c}$	$\hat{k}$	$\hat{c}$
MoM	1.993500197	2.980329991	1.993500205	4.329976906
L-MoM	1.977194146	2.979837573	1.977194154	4.329261496
ML	2.016383581	2.987563636	2.016383581	4.340486316

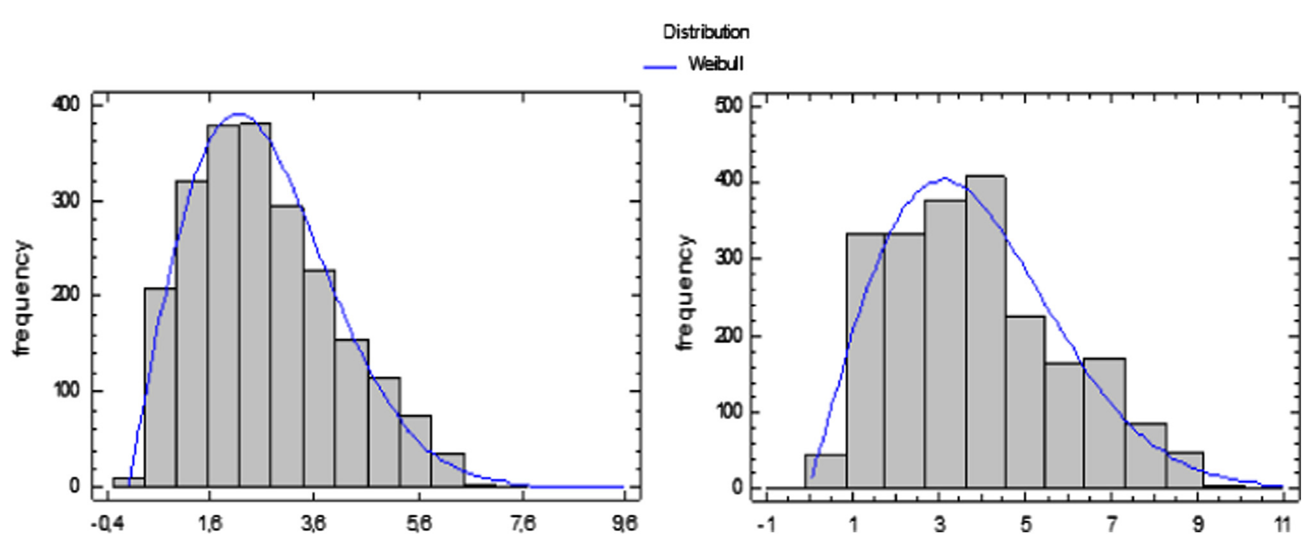

Fig. 2. Distribution of wind speed at 10 m and 30 m.

Table 3
Calculated power density values.

	MoM		L MoM		ML	
	10 m	30 m	10 m	30 m	10 m	30 m
Density value(P_w)	21.54	66.06	21.72	66.61	21.44	65.75

formula [29,30]:

$$P_w = \frac{1}{2}\rho c^3 \Gamma\left(1+\frac{3}{k}\right) \tag{16}$$

The standard air density value is calculated to be 1.22 at 15 °C and 1 atm pressure in dry air [31]. In the formula for estimated wind power density, the standard air density value has been taken to be $\rho = 1.22$. Moreover, the wind power density calculated according to the wind speed data and modeled in line with the three various methods is displayed in Table 3.

As evident from these findings, the calculated force density values based on the parameter estimations obtained from the various methods also prove to be different. Considering the findings in Tables 2 and 3 it is evident that the value of wind power density depends on the correct parameter estimation.

The findings show that, with large sample sizes, the ML method yields better results than the other methods. (This can be inferred from the simulation results presented in Section 3.) As the data set utilized in this study comprises 2200 observations, estimation via the ML method is most efficient in the calculation of the wind power density of the shape (k) and scale (c) parameter values.

6. Results and discussion

In this study, introducing L-MoM method and comparing it with other methods is aimed. So that in the literature, usage of L-MoM method, which gives more effective results with less data, could be extended. Correctly analysis of a small number of wind speed data that can be achieved with time and financial constraints can be ensured. It has been noted that in comparison to the other methods the L-MoM method yields better results with small sample sizes.

Table 1 shows that, as the sample size increases the ML method proves to be more efficient. Moreover, in this study, MoM, L-MoM and ML parameter estimation methods have been used in analyzing an actual data set. Wind power densities have also been calculated with the help of estimated parameter values. As observed in Table 3, wind force densities vary based on different parameter estimation methods. Considering that wind energy plants are built based on the calculated wind force densities, it is evident that the parameter estimation method is of great importance.

L-MoM method is more actual method than MoM and ML. But data analysis of wind speed is often done by help of parameters which are found by using MoM and ML estimators. Parameter estimate about wind speed data would be more accurate considering cases in which the optimum result is given from used parameter estimation methods. Instead of using the same method in different conditions to use the best method that matches the conditions provide calculated energy potential to be more accurate. L-MoM method allows estimating parameters more accurate with less observation (please see; Table 1 for MSE values about estimation methods).

Wind energy is an increasingly popular source of renewable energy. To use this energy effectively, some crucial steps should be considered. Initially, the fitting of wind speed data to a certain probability distribution function should be determined statistically. Then, parameters should be estimated using the most effective estimation method for a determined distribution. At this point, the best results can be achieved through administering not only one but, rather, multiple parameter prediction methods and choosing the most effective one. Finally, utilizing the values obtained from the most suitable parameter estimation method, wind power density potential can be calculated in an accurate and precise manner. The L-MoM method, which we presented in this study, may be beneficial for research using small sample sizes. Companies that want to benefit from wind speed could obtain information about appropriateness of the region to invest more reliably with less time and cost by L-MoM method.

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